

Calculation of the Bohr Magneton Using the Zeeman Effect

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Abstract

The Zeeman Effect was predicted by Hendrik Lorentz and first observed by Pieter Zeeman in 1896. It refers to the 'splitting' of atomic energy levels in the presence of a strong magnetic field. This splitting allows for a greater number of distinct energy level transitions to take place. These transitions can be observed as the splitting of spectral lines in light from an excited species, when that species is in the presence of a strong magnetic field. A Fabry-Perot etalon was used to generate an interference pattern, which was observed with a CCD camera. The geometry of the pattern was used to calculate the corresponding energy transition, and a hall probe was used to measure the B field. These quantities are related by the equation $\Delta E = \mu_B \cdot B$, and so were used to calculate the value of the Bohr Magneton, $\mu_B = 1.01 \pm 0.018 \times 10^{-23} \text{JT}^{-1}$.

I. INTRODUCTION

THE Zeeman Effect refers to the splitting of spectral lines from an excited species (and hence, of the energy level transitions that produce them) in the presence of a strong magnetic field. It was predicted by Hendrik Lorentz and first observed experimentally by the Dutch physicist Pieter Zeeman in 1896 [1].

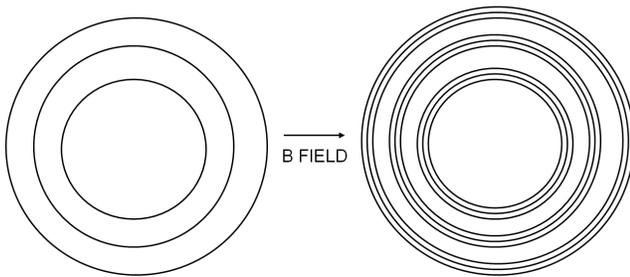


Figure 1: Typical splitting of an interference rings pattern from a Fabry-Perot etalon due to the Zeeman effect.

This interference pattern can be explained quantum-mechanically, in terms of the quantum numbers m and l - the Zeeman effect splits each energy level into $2l + 1$ separate levels. However, the actual electron energy level transitions which are allowed still follow the selection rule $\Delta m = 0, \pm 1$. For the transition we are interested in, this makes for a total of nine different transitions between the two levels. There are only three rings because many of the transitions actually have the same ΔE , even though they are from different Zeeman components.

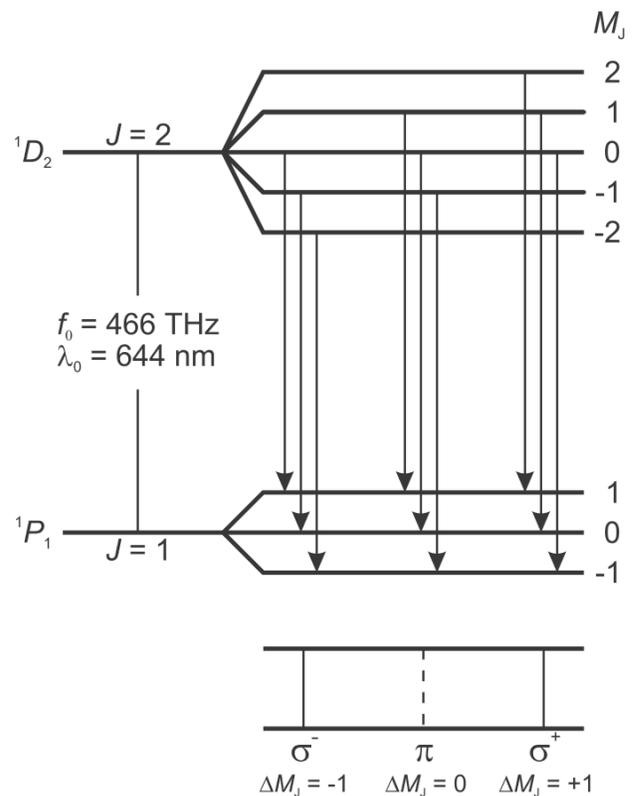


Figure 2: Energy level diagram showing the effect of a B-field on the possible energy levels and transitions for singlet state [2]

Although the splitting of energy levels is a purely quantum-mechanical process, it can be easily explained with a classical analog: imagine a charged particle, such as an electron, with a circular orbit. Then, apply a strong, homogeneous magnetic field. The electron feels a force proportional to $\vec{B} \times \vec{V}$, the cross product of its velocity with the field. This results in the frequency of the electron's orbit either increasing or decreasing, depending on the orientation of the magnetic field

relative to the electron orbit.

One more consequence of the Zeeman effect is the polarisation of the light emitted from the excited species. This is because the distribution of angular momentum for the emitted photons depends upon which energy level transition they have undergone. So one aim of the experiment will be to measure this polarisation and verify that it makes physical sense.

The Zeeman effect can also be used to calculate the value of μ_B , the Bohr Magneton, a physical constant which is used to express the dipole moment of electrons. It corresponds to the angular momentum of an electron in the lowest orbital. This constant linearly relates the change in energy levels to the strength of the magnetic field.

$$\Delta E = \mu_B \cdot B \quad (1)$$

The interference pattern in fig. 1 can be used to find the angular distances to the different rings, which are related to the geometric properties of the Fabry-Perot etalon and the energy of the transition by the following equation:

$$\Delta E \simeq \frac{-\alpha \Delta \alpha}{n^2} E \quad (2)$$

Where ΔE is the energy of the transition, α is the distance to the π ring with $B = 0$ from the center, $\Delta \alpha$ is the distance to the σ lines, E is the energy of the original transition and n is the refractive index of the lens.

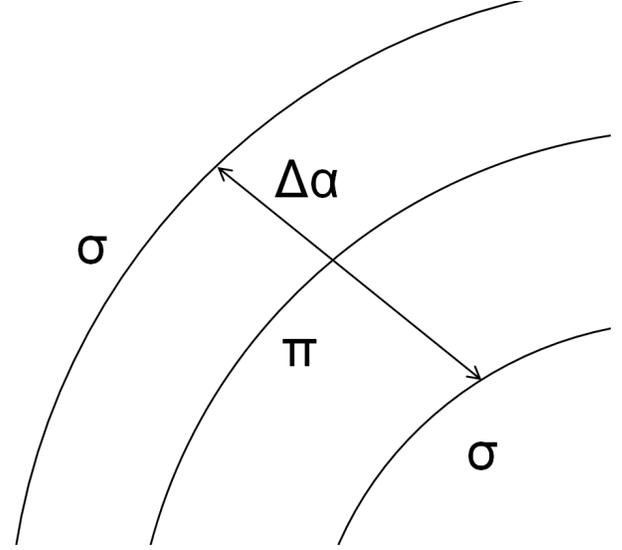


Figure 3: Calculation of $\Delta \alpha$ from the position of the rings. The π line is at position α from the center, and the σ lines are separated by an angular distance $\Delta \alpha$

An experiment was carried out to calculate the value of μ_B from the split of energy levels in a magnetic field of known strength, from an energy transition of known energy, and to observe the polarisation properties of light emitted from an excited species in a strong magnetic field.

II. EXPERIMENTAL METHODS

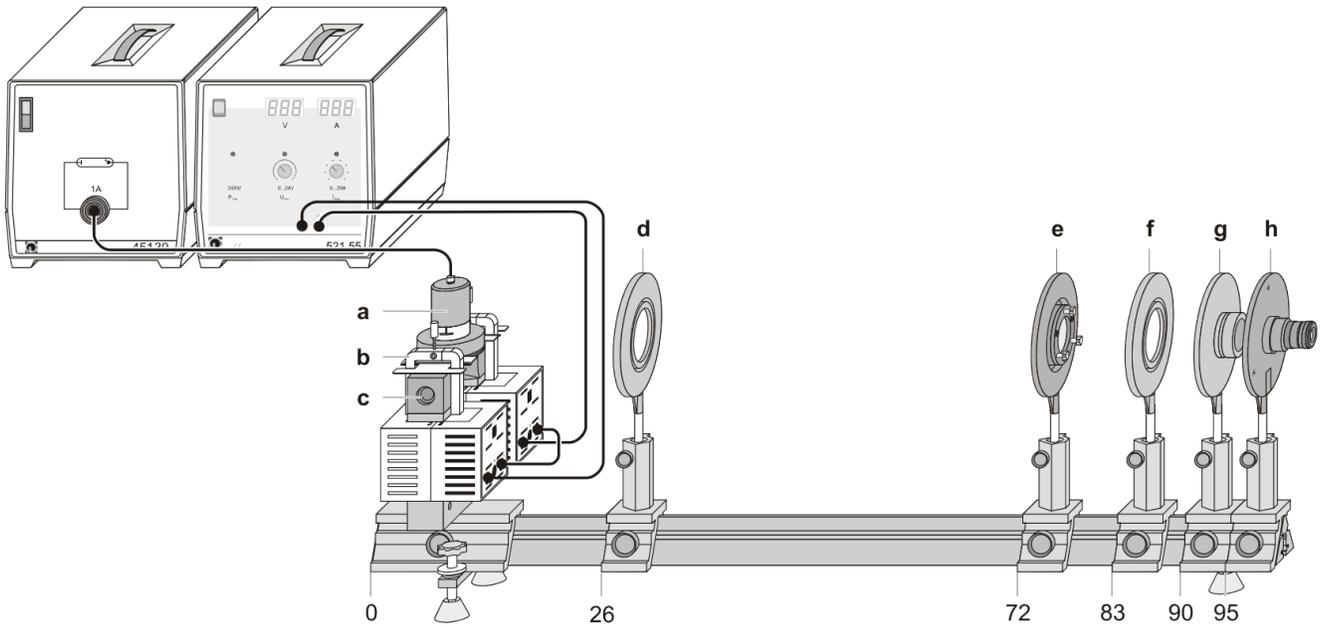


Figure 4: Diagram of the optical bench apparatus provided by LD Didactic [2]

The excited species, in this case, was a cadmium lamp [a], wherein the transition had a ΔE of 1.93eV . This lamp was clamped [b] to a strong electromagnet [c], which was connected to a power supply. The electromagnet produced a B field, which was approximately uniform across the Cd lamp, and whose direction could be altered. Additionally, varying the current and P.D. allowed the magnitude of the B field to be varied.

The lamp was positioned on an optical bench along with a series of other components, designed to produce the interference pattern.

When the lamp was turned on, the light emitted by the Cd lamp passed through a condensing lens [d], such that it was approximately parallel, then through the Fabry-Perot etalon [e], an imaging lens [f] with a known focal length (in this case, 150mm). Finally, the light passed through a red-coloured filter [g]. Although it is possible to use an ocular to observe the ring pattern directly, a CCD camera was used later in the experiment. Both the camera and ocular were positioned at the focal point of the lens. The data from the CCD was transferred to a computer via USB for analysis.

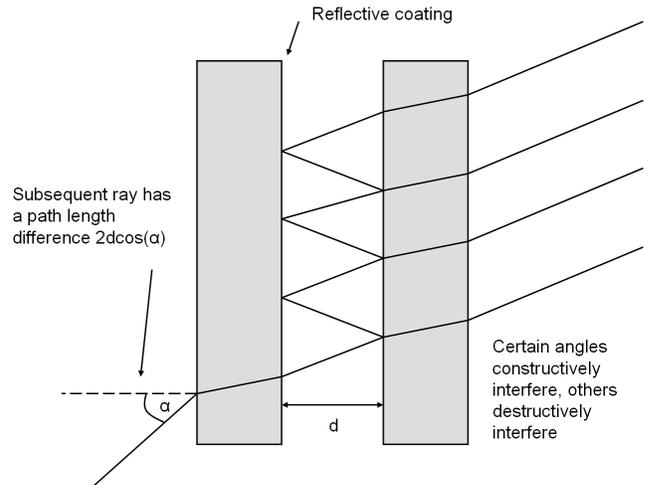


Figure 5: The Fabry-Perot etalon is responsible for producing the ring-like interference pattern from the Cd spectrum.

Optionally, a quarter-wave plate, polarising filter or both can be positioned in front of the etalon.

III. RESULTS & DISCUSSION

I. POLARISATION OF LIGHT

With a magnetic field perpendicular to the direction of the light, a polarising filter was introduced into the path of the light, and turned through a series of angles. It was observed that, at an angle of 0 degrees, only the σ lines were visible, but at 90 degrees only the π lines were visible. Therefore, the light from the σ and π lines are polarized orthogonally. Not only that, but it can also be deduced that light emitted from a source undergoing the Zeeman effect is polarised such that the π component is antiparallel to the magnetic field, and the σ component is parallel.

Then, the magnetic field was rotated so that it was parallel to the direction of the light. This time, the interference rings split into 2, not 3 - the σ rings were visible, but the π rings were not. To confirm this, the polarising filter was used again, but regardless of the direction of the field, pairs of dim σ lines were seen.

We know that the light must be polarised - but it's not linearly polarised, because it can be viewed from any polarisation angle - so it's reasonable to assume that, as the intensity is approximately halved, the light is circularly polarised instead. The intensity is halved because the angle of polarisation varies with time - the polarisation angle is the same as the filter angle exactly half of the time).

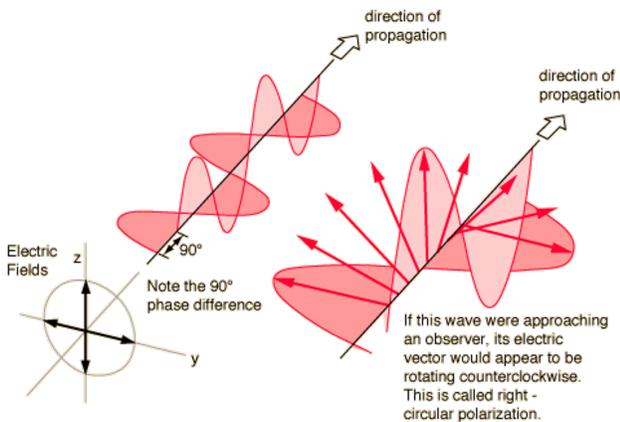


Figure 6: Circular polarisation of light [3]

Circularly polarised light is essentially two polarised light rays in two orthogonal directions, with a $1/4\pi$ phase difference between them. So, if we were to introduce a quarter-wavelength plate, it should be possible to configure them such that the phase difference between the different directions is removed. This results in the circularly polarised light being changed into linearly polarised light with both an x and a y

component. It can then be verified that the light is orthogonally polarised using a polarising filter.

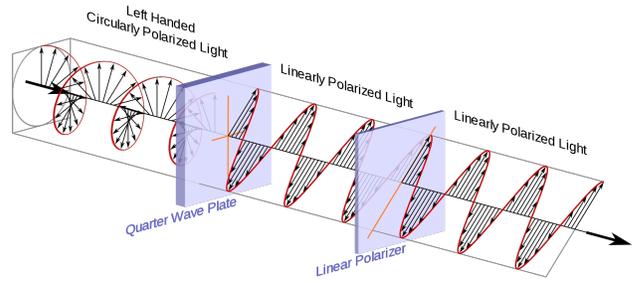


Figure 7: Effect of a quarter-wave plate and polarising filter on linearly polarised light. [4]

A quarter-wave plate was placed into the path of the light, before the polarising filter. It was found that one of the σ lines was polarised in the x direction, and the other was polarised in the y direction. So at an angle of -45 degrees, the inner line was visible, at 0 they were both visible, and at 45 degrees the outer line was visible. In other words, the circularly polarised light had become linearly polarised again.

II. ESTIMATION OF THE BOHR MAGNETON

We know that the Bohr magneton linearly relates the strength of the magnetic field B to ΔE . However, the nature of the apparatus means that both of these cannot be measured directly at the same time, as the area in which the homogeneous magnetic field is created is very small. So, the lamp was temporarily removed, and a hall probe was inserted into its place. This allowed for a calibration curve to be plotted, relating the strength of the field to the current supplied to the magnet.

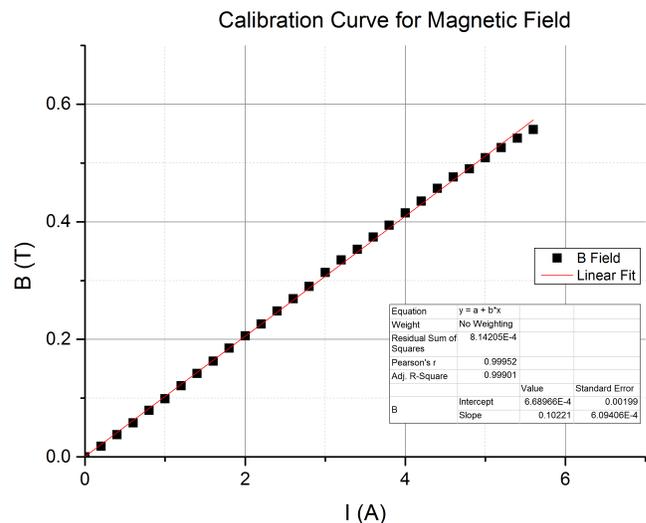


Figure 8: Calibration curve showing how B varies with I , under a constant current, with a linear fit applied in Origin.

Because N and E are both known, it should be possible to calculate ΔE provided we have the values of α and $\Delta\alpha$. For this, a CCD was used, which measured the positions of the rings from the diffraction pattern as a 1-D series of intensity peaks.

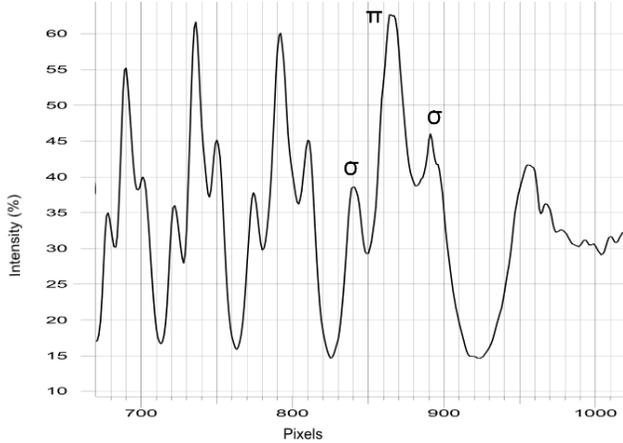


Figure 9: Typical image from the CCD camera, showing several series π and σ lines.

Note that the x-axis of this graph is in pixels, rather than the angular distance that we want. However, the pixel size was given by the manufacturer as $14\mu m$. Because the distance from the CCD to the lens was also known, the linear distance can easily be converted to an angular distance using the relation:

$$\alpha = \arctan\left(\frac{x}{d}\right) \quad (3)$$

Where α is the angle, x is the linear distance and d is, in this case, the focal length of the lens ($150mm$).

The positions of the innermost set of rings were recorded for a range of B fields, with between $2A$ and $5A$ of current flowing through the magnet. The calibration curve was used to calculate B , and equations (2) and (3) were then used to calculate ΔE for each value of B . Finally, the results were plotted on a graph, and a linear curve fit was applied in the software package 'Origin'. This gave a value for the Bohr Magneton of $\mu_B = (1.01 \pm 0.018) \times 10^{-23} JT^{-1}$.

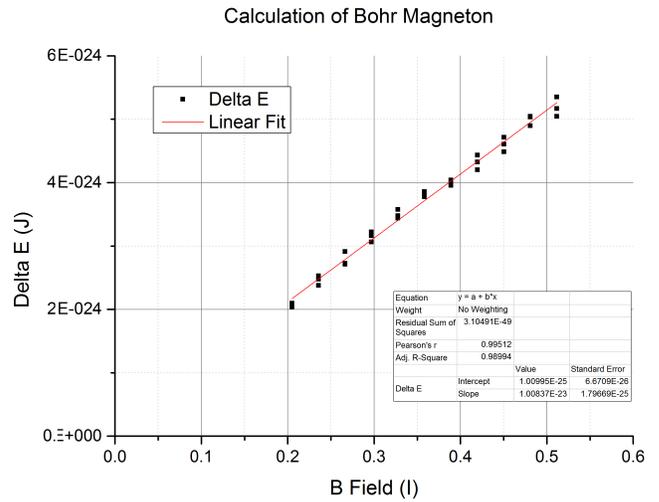


Figure 10: Linear fit used to calculate the Bohr Magneton

This experimentally determined value of B was a slight overestimate (109% of the accepted value, $9.27 \times 10^{-24} J \cdot T^{-1}$ [5]), the small associated error means that the deviation from the accepted value is not the result of random error. So, which systematic errors are associated with this method, and how could they have resulted in an overestimate?

Firstly, the calculation of energy shift is only an approximation - it assumes that our angles are small, so it approximates both $\sin(\alpha) \simeq \alpha$ (the larger the α , the larger the overestimate), and an infinitesimal change in the photon wavelength $d\lambda$ to a small change in λ . So it makes sense that the calculated B is an overestimate.

Secondly, there was a small systematic error in the calibration calculation of the B field across the lamp. Theoretically, the calibration curve should have a zero y-intercept, but realistically there was a small, positive y-intercept.

Finally, the angular distance relation makes certain assumptions about the geometry of the setup - namely, the light sensor in the CCD is exactly perpendicular to the direction of the incident light, and the sensor inside the CCD is at exactly the focal point of the lens opposite. In reality, neither of these things were precisely true, which resulted in the angles towards one end of the light sensor being a slight overestimate, and angles on the other side being a slight underestimate. This effect was actually observed on the intensity spectrum, as one side of the CCD was measuring higher intensities than the other.

IV. REFERENCES

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- [5] Peter J. Mohr, Barry N. Taylor, and David B. Newell. Codata recommended values of the fundamental physical constants: 2006. *Rev. Mod. Phys.*, 80.